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2023-2024 Math League Preliminary Round

(Grade 6)

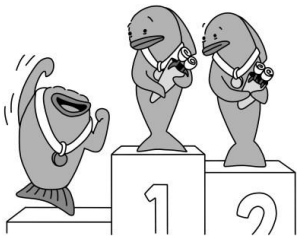
(Unofficial version, for reference only)

1. Which of the following can be written as the sum of the reciprocals of two consecutive integers?

   A) $\frac{1}{20}$ B) $\frac{7}{20}$ C) $\frac{9}{20}$ D) $\frac{19}{20}$

2. Mia is the 3rd-fastest fish in her bowl. If there are 9 fish in the bowl, what percent of the **other** fish swim faster than Mia?

   A) 25 B) 30 C) $33\frac{1}{3}$ D) $37\frac{1}{2}$



3. How many factors of 2 does 20 000 have?

   A) 1 B) 5 C) 9 D) 10

4. If the opposite faces of a rectangular box are congruent rectangles, what is the greatest possible number of different edge-lengths?

   A) 2 B) 3 C) 4 D) 8

5. My grocery bag is strong enough to hold 8 of my oranges or 12 of my lemons. If I put 2 of my oranges in the bag, how many of my lemons could the bag hold?

   A) 6 B) 8 C) 9 D) 10



6. In my town it rains every third day. What is the greatest number of rainy days possible in a month?

   A) 9 B) 10 C) 11 D) 12

7. When I add 7 or subtract 13 from my quiz grade, I get a perfect square. The sum of the digits of my quiz grade is

   A) 5 B) 7 C) 9 D) 11

8. All 189 cars in a parking lot are either red or black, and the ratio of red cars to black cars is 8:13. How many of the 189 cars are black?

A) 15 B) 45 C) 72 D) 117

9. At the Bear Fair, the number of fish in Grizzly Greta's bucket is a prime number. If I remove a prime number of fish, the number of fish remaining **cannot** be

A) 1 B) 4 C) 7 D) 9

10. Ann, Bill, and Carl each ate a popsicle last Sunday. If Ann eats a popsicle every 4 days, Bill eats one every 5 days, and Carl eats one every 6 days, when will they next eat popsicles on the same day?

A) Monday B) Tuesday C) Wednesday D) Thursday

11. If the angles of an acute triangle all have integral measures, what is the least possible measure of an angle in the triangle?

A) 1° B) 2° C) 3° D) 4°

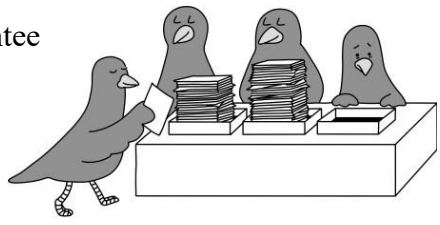
12. What is the least positive integer such that the product of this integer and 480 is a perfect square?

A) 15 B) 30 C) 120 D) 480

13. For how many different integer side-lengths of a square is the area of the square numerically less than the perimeter of the square?

A) 2 B) 3 C) 4 D) 5

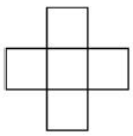
14. Three pigeons in a flock are running for Head Pigeon. How many votes out of 300 possible votes must a pigeon receive to guarantee that it receives the most votes?



A) 100 B) 101

C) 150 D) 151

15. How many different quadrilaterals are in the figure shown?



A) 7 B) 9 C) 10 D) 11

16. How many different isosceles triangles with integer side-lengths have perimeter 28?

A) 8 B) 6 C) 5 D) 4

17. In the year 2025, the sum of the three leftmost digits (2 + 0 + 2) is less than the ones digit (5). For how many numbered years in the 21st century is this true?

A) 21 B) 24 C) 26 D) 28

18. How many different values can the perimeter of a rectangle with integer side-lengths and area 48 have?

A) 5 B) 6 C) 8 D) 10

19. Shana put her 15 gumballs into 3 bags, so that each bag had a different number of gumballs. What is the greatest number of gumballs possible in the smallest pile?



A) 1 B) 2 C) 4 D) 6

20. What is the sum of the digits of the greatest integer less than 100 that has a remainder of 2 when divided by 5, and a remainder of 3 when divided by 8?

A) 13 B) 14 C) 16 D) 17

21. If I have 4 quarters, 6 dimes, and 8 nickels, in how many different ways can I make 1 dollar?

A) 10 B) 11 C) 12 D) 13

22. How many positive integers less than 100 have an even number of factors?

A) 90 B) 80 C) 70 D) 60

23. Which expression has the greatest value?

A) 2^{500} B) 3^{400} C) 5^{300} D) 7^{200}

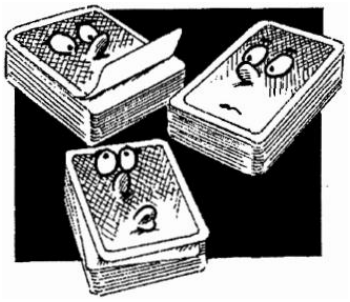
24. The remainder when $7^{77} + 5^{55} + 3^{33} + 2^{22} + 1^{11}$ is divided by 2 is

A) 5 B) 4 C) 1 D) 0

25. Which of the following is closest in value to 0.8^{88}?

A) 0 B) 1 C) 8 D) 64

26. After I wrote each of the first n positive integers on a different card, I split these cards into three piles (which may or may not contain different numbers of cards). For how many distinct values of $n \leq 2022$ can the numbers written on the cards in each pile have the same sum?



27. A sequence consists of only 0s, only 1s, or both 0s and 1s. Any 0 in the sequence must be one of an odd number of consecutive 0s. Any 1 in the sequence must be one of an odd number of consecutive 1s. How many such different ten-term sequences are possible?

    Note: The only possible 1-term sequences are 0 and 1. The only possible 2-term sequences are 01 and 10. The only possible 3-term sequences are 000, 010, 111, and 101.

28. Altogether, three different positive integers have 7 digits, not all different. One of the numbers is the average of the other two. What is the largest possible value of this average?

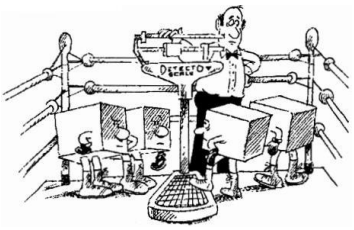
29. What is the largest possible quotient of a positive 3-digit number divided by the sum of its digits, if the 3 digits must be different?

30. When $n > 1$ is an integer, $n!$ represents the product of the integers from 1 through n. What are the only two integers n for which the product $48! \times n!$ is the square of an integer?

    Note: Please enter the sum of the two integers as your answer.

31. If $\frac{x}{x+2} = \frac{2022}{2023}$, what is the value of $\frac{x+1}{x+2}$?

32. Four cubes look identical, but their weights are in the continued ratio 2:4:6:9. Using only a balance scale, what is the least number of balancings that would be needed to identify the heaviest cube?

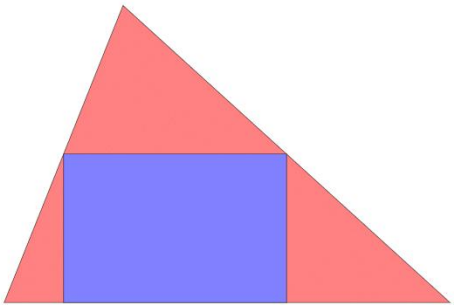


33. For what value of A will a circle of area A quadruple its area by increasing the length of the circle's radius by 1?

    Note: A can be written as $k\pi$, that is $A = k\pi$. Please enter the value of k as your answer.

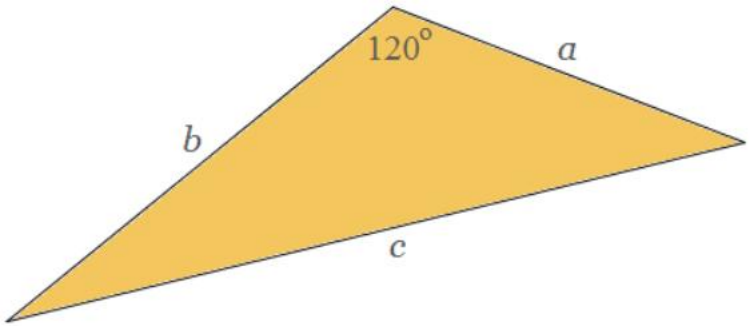
34. What is the area of the largest rectangle inscribed in a triangle with side lengths 8, 15, and 17?

    Note: The figure below shows a rectangle inscribed in a triangle.



A) 30 B) 40 C) 50 D) none of above

35. In the following triangle (not drawn to scale), $a = 5$, $b = 16$, how long is c?



A) 17 B) 18 C) 19 D) none of above

Answers										
Question	1	2	3	4	5	6	7	8	9	10
Answer	**C**	**A**	**B**	**B**	**C**	**C**	**D**	**D**	**C**	**D**
Question	11	12	13	14	15	16	17	18	19	20
Answer	**B**	**B**	**B**	**D**	**D**	**B**	**D**	**A**	**C**	**A**
Question	21	22	23	24	25	26	27	28	29	30
Answer	**D**	**A**	**C**	**D**	**A**	**1346**	**110**	**504**	**91**	**97**
Question	31	32	33	34	35					
Answer	$\frac{4045}{4046}$	**2**	**1**	**A**	**C**					