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2023-2024 Math League Preliminary Round

(Grades 10-12)

(Unofficial version, for reference only)

1. The difference between two 3-digit primes **cannot** be a

   A) multiple of 7 B) perfect square

   C) perfect cube D) 2-digit prime

2. Jo has the same number of math awards as the positive difference between the two roots of $x^2 - 5x - 36 = 0$. She has _?_ math awards.



   A) 4 B) 5

   C) 9 D) 13

3. Di and her daughter Vi both had a birthday today. If Di turned 37 and Vi turned 4, how old will Di be on the day Vi becomes exactly one-fourth of Di's age?



   A) 7 B) 11 C) 44 D) 48

4. Diego spent d dollars on a c crates of tennis balls. If each crate contains b balls, how many cents did each tennis ball cost Diego?

   A) $\frac{d}{cb}$ B) $\frac{cb}{d}$ C) $\frac{cb}{100d}$ D) $\frac{100d}{cb}$

5. If the x- and y-intercepts of a line have non-zero values and the sum of the intercepts is zero, then the slope of the line must be

   A) 0 B) positive C) negative D) undefined

6. Steeler's Wheels was stuck in the middle of a sales slump with u unicycles, b bicycles, and t tricycles, so they slashed prices and everything sold! If there was a \$3 per-wheel tax on all sales, how many dollars in total were the wheel taxes?



A) $3(3t + 2b + u)$ B) $3tbu$

C) $3(t + b + u)$ D) $3(t + 2b + 3u)$

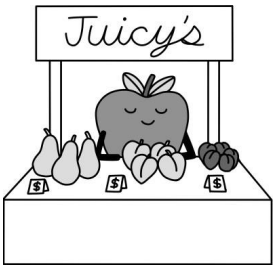
7. If 10% of a is equal to 20% of b, then $2b =$

A) $\frac{a}{2}$ B) a C) $2a$ D) $4a$

8. My 21 coins, all nickels and dimes, total \$1.40. I have _?_ nickels.

A) 5 B) 7 C) 8 D) 14

9. At Juicy's fruit stand, 2 peaches cost as much as 3 pears, 2 plums cost \$3 less than 3 peaches, and 3 pears cost as much as 4 plums. How much do 2 peaches cost?



A) \$1.50 B) \$2.00 C) \$3.00 D) \$4.00

10. $|2r - 3s - 4t| =$

A) $|4t + 3s - 2r|$ B) $|4t - 3s - 2r|$

C) $|-2r - 3s - 4t|$ D) $|2r + 3s + 4t|$

11. The ratio of cats to birds on my street at noon was 3:8, but then 3 new cats were born, scaring away 4 of the birds and leaving a cat-to-bird ratio of 3:5. How many birds were on my street at noon?



A) 8 B) 16 C) 24 D) 32

12. The least positive integer that is the square, cube, 4th, and 5th power of other integers has _?_ positive divisors.

A) 14 B) 28 C) 60 D) 61

13. Myla's sketch of a house is an x cm by y cm rectangle that has a common side of length x cm with a triangle of height of y cm. She colored in both polygons with blue crayon, except that within the x cm by y cm rectangle she didn't color in the two $\frac{x}{4}$ cm by $\frac{y}{4}$ cm rectangles she

drew as windows and an $\frac{x}{4}$ cm by $\frac{y}{2}$ cm rectangle she drew as a door. Myla colored in an area of _?_ square cm.

A) $\frac{5xy}{4}$ B) $\frac{21xy}{16}$ C) $\frac{7xy}{4}$ D) $\frac{29xy}{16}$



14. My locker combination requires dialing in order three 2-digit numbers under 50, including one prime, one even number, and one odd multiple of 3. How many such combinations are possible?

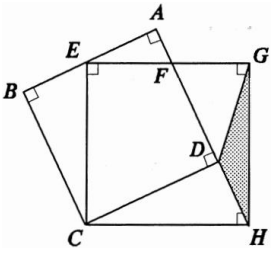
A) 1320 B) 2520 C) 3960 D) 7920

15. If $f(x, y) = x^2 + y^3$, $g(x, y) = (2x, 3y)$, $f(a, b) = 33$, $a > 0$, and $f(g(a, b)) = 316$, then $a + b =$

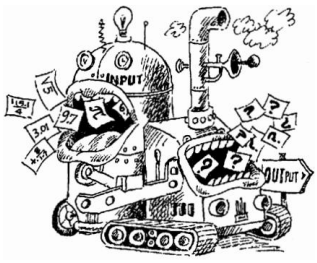
A) 5 B) 7 C) 12 D) 24

16. If $x = 3^{2023}$ and $y = x^{2024}$, the ones digit of y^{2025} is

A) 1 B) 3 C) 7 D) 9



17. In the accompanying diagram, the area of square $ABCD$ is 600, E is the midpoint of $\overline{AB}$, $CEGH$ is a square, and points A, F, D, and H are collinear. What is the area of $\triangle DGH$, shown shaded?



18. Let $[x]$ represent the greatest integer $\leq x$. Thus, $[3.2] = 3$. A machine takes an input x, produces the output $x[x]$, and uses this output as the new input. If the first input is x_0, the first output is called x_1, and x_1 is used as the second input, etc. If $x_0 = 2.65$, what is the value of x_3?

19. Each of a, b, c, and d has a value chosen independently, with equal probability $\frac{1}{10}$, from {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. The values chosen need not be different. What is the probability that $ab + cd$ is odd?

20. It can be proven that $4^{2000} + 4^x + 4^{2023}$ is the square of an integer for exactly 3 integer values of x. What is the largest such value of x?

21. What is the smallest positive integer n for which a square of area $2023 + n$ has integer side-lengths?

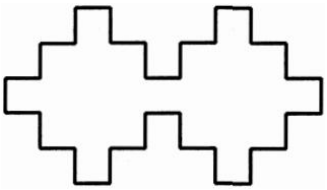
22. If the circumference of a circle is increased by 10%, by what per cent is the area of the circle increased?

    Note: This is how to enter your answer. For example, if your answer is 30%, please enter 30.

23. Alan can paint my house in a days, while Bob can paint it in b days. Working together, each at his usual rate, Alan and Bob can paint my house in c days. If a, b, and c are different integers, and $abc = 36$, what is the value of c?



24. The sides of the 36-gon at the below are all congruent and all either horizontal or vertical, as shown. If the 36-gon's perimeter (in units) is equal to its area (in square units), what is the 36-gon's perimeter?



25. What is the smallest composite number > 1 that is not divisible by any of 2, 3, 5, or 7?

26. For what integer a is $x^{13} + x + 90$ divisible by $x^2 - x + a$ as polynomials?

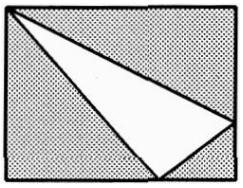
27. What is the smallest perimeter of a triangle the length of each of whose sides is the square of a different integer?

28. What are the only two pairs of positive integers (x, y) that satisfy

$$x^y - y^x = 1?$$

    Note: This is how to enter your answer. If your two ordered pairs are (x_1, y_1) and (x_2, y_2), please enter the value of $x_1 + y_1 + x_2 + y_2$.

29. A 9 × 12 rectangle is split into four triangles, and then three of the four triangles are shaded, as shown. If the area of each of the two larger shaded triangles is one-third of the area of the rectangle, what is the area of the unshaded triangle?



30. What is the ordered pair of single-digit integers (b, c) for which my calculator's ten-digit display lists the solutions of $3x^2 + bx + c = 0$ as 1.535183758 and -0.868517092?

    Note: This is how to enter your answer. If your ordered pair is (b, c), please enter the value $c - b$.



Answers										
Question	1	2	3	4	5	6	7	8	9	10
Answer	**D**	**D**	**C**	**D**	**B**	**A**	**B**	**D**	**C**	**A**
Question	11	12	13	14	15	16	17	18	19	20
Answer	**C**	**D**	**A**	**D**	**B**	**A**	**75**	**689**	$\frac{3}{8}$	**2045**
Question	21	22	23	24	25	26	27	28	29	30
Answer	**2**	**21**	**2**	$51\frac{21}{25}$	**121**	**2**	**77**	**8**	**30**	**-2**