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2023-2024 Math League Preliminary Round

(Grades 8-9)

(Unofficial version, for reference only)

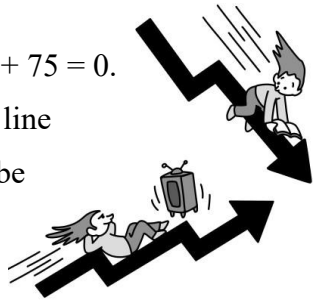
1. $(x-1)(x-2)(x+3)(x+4) = (x^2+3x-4) \times (\ ?\ )$

   A) x^2+3x-4 B) x^2+2x-3 C) x^2+2x-8 D) x^2+x-6

2. If the average of v and w is 16 and the average of x, y, and z is 26, what is the average of v, w, x, y, and z?

   A) 42 B) 22 C) 21 D) 20

3. A graph of my algebra grades this year forms a line with equation $5x - y + 75 = 0$. If the graph T of the amount of time I spend watching television forms a line perpendicular to the graph of my grades, the equation for graph T could be



   A) $5x + y - 75 = 0$ B) $5x - y - 75 = 0$

   C) $x - 5y + 15 = 0$ D) $x + 5y - 15 = 0$

4. Moving at x cm per second is the same as moving at __?__ m per hr.

   A) $36x$ B) $0.6x$ C) $\frac{5x}{3}$ D) $\frac{x}{36}$

5. If $a \bigstar b = (b-a)^2 - 2(b-a) + 1$, then what is the value of $20 \bigstar 24$?

   A) 9 B) 10 C) 14 D) 15

6. If I square Ed's age, I get the same number I get when I multiply his age by 12 and subtract 35. The same is true of Dee's age, but Ed and Dee are not the same age. What is the sum of Ed's and Dee's ages?

   A) 12 B) 23 C) 35 D) 47

7. If $\frac{1}{3}$ of the number of cats in a pet store is equal to $\frac{1}{4}$ of the number of dogs, and $\frac{1}{6}$ of the number of dogs is equal to $\frac{1}{7}$ of the number of fish, then the ratio of fish to cats in the store is

A) 9:10 B) 10:9 C) 7:3 D) 14:9

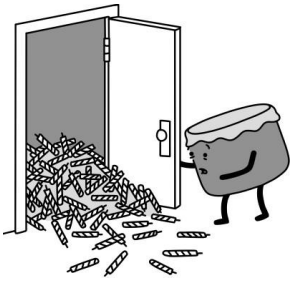
8. On April 1, 2024, El was 5 times as old as Mel was on April 1, 2000. If El is exactly 4 years older than Mel, what will be the sum of Mel's age and El's age on April 1, 2030?

A) 28 B) 48 C) 58 D) 78

9. If $x^{333} - 8^{444} = 0$, then $x =$

A) 5 B) 16 C) 32 D) 44

10. On every one of Ron's birthdays, his birthday cake has had a number of candles on it equal to Ron's age in years. Ron has now had a total of 120 candles on his birthday cakes. How many years old is Ron?



A) 10 B) 12

C) 15 D) 18

11. What is the difference between the two solutions $|10 - 4x|$ to = 12?

A) 2 B) 5 C) 6 D) 11

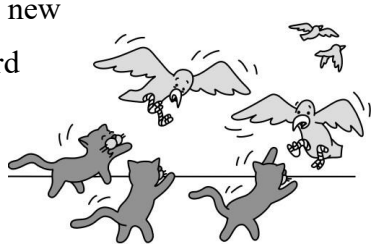
12. Caleb calls his cousins' cats to come consume a crate of cod. Caleb counts his catch, and comes up with a 2-digit number with the ones digit 5 greater than the tens digit. Caleb can calculate that reversing the digits creates a number that is 7 more than twice the original number. The sum of the 2 digits in the original number is

A) 7 B) 9 C) 11 D) 13

13. Sanford and his son sorted some junk. Sanford worked for 2 days alone, then his son joined him, and together they finished the job in 2 more days. Sanford alone would have needed 8 days to do the whole job, so his son alone would have needed _?_ days.

A) 2 B) 4 C) 6 D) 8

14. The ratio of cats to birds on my street at noon was 3 : 8, but then 3 new cats were born, scaring away 4 of the birds and leaving a cat-to-bird ratio of 3 : 5. How many birds were on my street at noon?



A) 8 B) 16

C) 24 D) 32

15. The least positive integer that is the square, cube, 4th, and 5th power of other integers has _?_ positive divisors.

A) 14 B) 28 C) 60 D) 61

16. Myla's sketch of a house is an x cm by y cm rectangle that has a common side of length x cm with a triangle of height of y cm. She colored in both polygons with blue crayon, except that within the x cm by y cm rectangle she didn't color in the two $\frac{x}{4}$ cm by $\frac{y}{4}$ cm rectangles she drew as windows and an $\frac{x}{4}$ cm by $\frac{y}{2}$ cm rectangle she drew as a door. Myla colored in an area of _?_ square cm.

A) $\frac{5xy}{4}$ B) $\frac{21xy}{16}$ C) $\frac{7xy}{4}$ D) $\frac{29xy}{16}$



17. My locker combination requires dialing in order three 2-digit numbers under 50, including one prime, one even number, and one odd multiple of 3. How many such combinations are possible?

A) 1320 B) 2520 C) 3960 D) 7920

18. If $f(x, y) = x^2 + y^3$, $g(x, y) = (2x, 3y)$, $f(a, b) = 33$, $a > 0$, and $f(g(a, b)) = 316$, then $a + b =$

A) 5 B) 7 C) 12 D) 24

19. If $x = 3^{2023}$ and $y = x^{2024}$, the ones digit of y^{2025} is

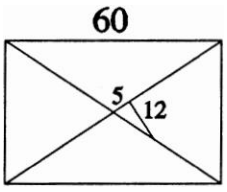
A) 1 B) 3

C) 7 D) 9



20. Four girls take turns flipping a fair coin until a head appears. What is the probability that the first girl is the first to flip a head?

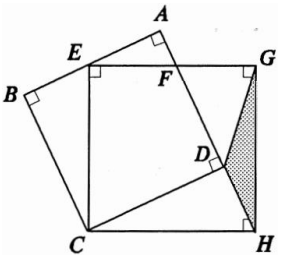
21. From a point on one diagonal of a rectangle of longer dimension 60, a line segment drawn perpendicular to the other diagonal creates a right triangle with leg-lengths 5 and 12, as shown. How long is a shorter side of the rectangle?



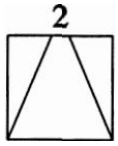
22. The sum of 3 consecutive terms of a geometric sequence is 62. If the sum of the common logarithms (*i.e.*, the base ten logarithms) of these 3 terms is 3, what is the value of the greatest of these 3 terms?

23. In acute triangle ABC, altitudes $\overline{BP}$ and $\overline{CQ}$ are drawn. If $AP = x$, $PC = 2x - 6$, $AQ = 8$, and $QB = x - 5$, what is the value of $\cos A$?

24. In the accompanying diagram, the area of square $ABCD$ is 600, E is the midpoint of $\overline{AB}$, $CEGH$ is a square, and points A, F, D, and H are collinear. What is the area of $\triangle DGH$, shown shaded?



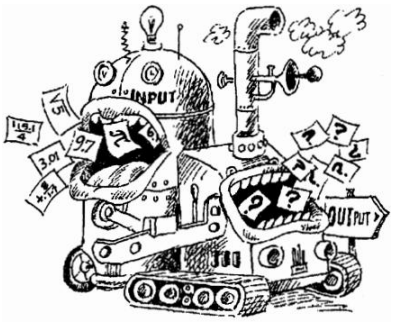
25. One base of an isosceles trapezoid is also a side of a square. The other base, whose length is 2, is on the square's opposite side, as shown. If the square's area is 144, what is the trapezoid's perimeter?



26. What is the perimeter of a right triangle if the length of its hypotenuse is 29 and the lengths of both legs are integers?

27. What is the least common multiple of 2^2, 4^2, 6^2, and 8^2?

28. Let $[x]$ represent the greatest integer $\leq x$. Thus, $[3.2] = 3$. A machine takes an input x, produces the output $x[x]$, and uses this output as the new input. If the first input is x_0, the first output is called x_1, and x_1 is used as the second input, etc. If $x_0 = 2.65$, what is the value of x_3?



29. Each of a, b, c, and d has a value chosen independently, with equal probability $\frac{1}{10}$, from {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. The values chosen need not be different. What is the probability that $ab + cd$ is odd?

30. It can be proven that $4^{2000} + 4^x + 4^{2023}$ is the square of an integer for exactly 3 integer values of x. What is the largest such value of x?

Answers										
Question	1	2	3	4	5	6	7	8	9	10
Answer	**D**	**B**	**D**	**A**	**A**	**A**	**D**	**D**	**B**	**C**
Question	11	12	13	14	15	16	17	18	19	20
Answer	**C**	**C**	**B**	**C**	**D**	**A**	**D**	**B**	**A**	$\frac{8}{15}$
Question	21	22	23	24	25	26	27	28	29	30
Answer	**40**	**50**	$\frac{2}{3}$	**75**	**40**	**70**	**576**	**689**	$\frac{3}{8}$	**2045**